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A HYDRODYNAMIC MODEL FOR NUCLEATE POOL BOILING

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Abstract—A hydrodynamic model of stagnation flow is proposed for saturated nucleate boiling over a flat surface. Through the established analytical results in axisymmetrical stagnation flow, a relation between the heat-transfer coefficient and the thermal boundary-layer thickness induced by rising bubbles is obtained, and a good agreement with measured results in the low heat-flux region is indicated. The predicted heat-transfer result is given as

$$q = c\,Pr^{0.33}\,k\,n^{0.5}\,\Delta T$$

where c is a numerical constant equal to 61·3 as determined from the boiling data of water. The predicted relation is found to be in agreement with boiling data of most liquids.

NOMENCLATURE

h_x,	local heat-transfer coefficient at x, Btu/ft^2 h °F;
$\bar{h}$,	average heat-transfer coefficient, Btu/ft^2 h °F;
k,	thermal conductivity of the liquid, Btu/ft h °F;
n,	active nucleation site density, ft^{-2};
Nu_x,	Nusselt number based on local heat-transfer coefficient at x, dimensionless;
$\overline{Nu}$,	Nusselt number based on average heat-transfer coefficient, dimensionless;
Pr,	Prandtl number of the liquid, dimensionless;
q,	average heat flux, Btu/ft^2 h;
Re_x,	Reynolds number at x, dimensionless;
s,	center-to-center distance of bubbles, ft;
u_∞,	x-component velocity at the outer edge of hydrodynamic boundary layer, ft/s;
x,	radial co-ordinate, ft;
y,	vertical co-ordinate to the heat surface, ft.

Greek symbols

α,	stagnation flow constant defined in (3), s^{-1};
β,	universal constant defined in (12), dimensionless;
γ,	constant defined in (16), dimensionless;
δ,	hydrodynamic boundary-layer thickness, ft;
δ_{th},	thermal boundary-layer thickness, ft;
ν,	kinematic viscosity of the liquid, ft^2/s.

INTRODUCTION

THE application of nucleate-boiling heat transfer to heat-removal problems in nuclear reactors and other contemporary equipment has prompted an increasing number of studies of the boiling phenomenon. Generally, such studies have provided either experimental data of a gross kind (such as average heat-transfer coefficients or heat fluxes) or they have provided both measurements and analyses to account for certain details of bubble nucleation, growth and collapse. While it is generally agreed that "bubble stirring action" is in some way the cause of increased heat transfer, the mechanism by which bubble action determines the heat flux is not clearly understood. Several mechanisms have been proposed (for a review and discussion of these mechanisms, see [1]) but none has led to the quantitative formulation of a heat-transfer prediction. In some recent works [2, 3] the discussion of the heat-transfer mechanism has been directed to the role of the thermal boundary layer over the heated surface, but little

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has been said about the flow mechanism in nucleate pool boiling.

The concept that boiling heat transfer is achieved through a hydrodynamic boundary-layer mechanism, was advanced heuristically by Gregorig [4] and independently by Zuber [5]. The existence of a thermal boundary layer in nucleate pool boiling was first indicated by the measurements of temperature distribution near the wall by Gunther and Kreith [6]. The variation of thermal boundary-layer thickness with heat-transfer coefficient was experimentally investigated by Yamagata and his co-workers [7] by means of optical measurements. The first attempt to predict boiling heat transfer through a hydrodynamic boundary-layer model was made by Zuber [5]. He assumed a similarity between nucleate-boiling heat transfer and heat transfer in laminar boundary-layer flow over a flat plate, and employed a bubble Reynolds number in place of the hydrodynamic Reynolds number. This led to a relation of the form

$$q \sim \Delta T^{a}\, n^{b}.$$

Such a relationship is reasonable from a physical standpoint since it relates q to surface condition —a variable often neglected in boiling heat-transfer predictions. Yamagata *et al.* [7] as well as Kurihara and Meyers [8] succeeded in correlating experimental data with the above equation, although their values of a and b differ from one another and from Zuber's values.

The present work intends to advance a new hydrodynamic model for the mechanism of nucleate-boiling heat transfer. Based on this mechanism, a simple quantitative heat-transfer prediction can be made in terms of the physical properties of the boiling liquid and the active nucleation site density. While idealizations are made for the physical model, quantitative results predicted show a good agreement with most of the measurements [7–9].

STAGNATION-FLOW MODEL

Consider now the saturated nucleate boiling over a flat surface. The viscous shear between rising bubbles and surrounding liquid induces a flow field in the liquid phase as shown in Fig. 1(a). While it is known that the active nucleation sites are distributed rather randomly over the heating surface, it will be assumed, as it has been in several previous works [3, 5], that on the average the influence domain of a single bubble is given by

$$s^2 = n^{-1}. \tag{1}$$

The average center to center spacing of bubbles thus becomes

$$s = n^{-0\cdot5}. \tag{2}$$

As the flow and heat-transfer characteristics in the influence domain of a single bubble will serve to characterize the flow and heat transfer at any point on the heating surface, the gross boiling heat transfer can then be determined.

The flow and heat transfer in the influence domain of one bubble may be described if two regions in nucleate pool boiling are considered separately. In the first region, the active site density is small, the spacing between bubbles is large as compared to the bubble diameter, and the mutual influence between bubbles is negligibly small. This region is called the region of isolated bubbles [5]. The experimental

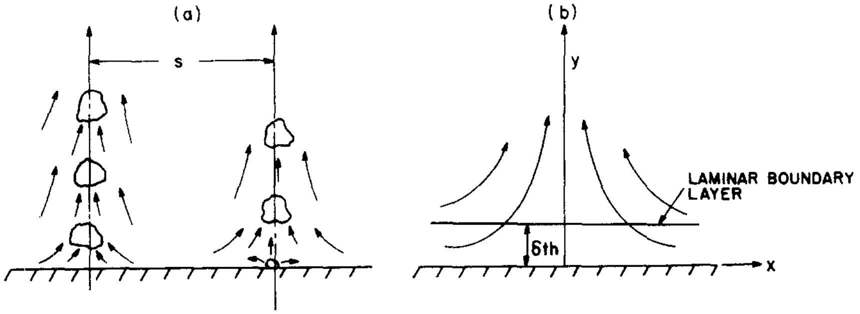

FIG. 1. Actual and idealized flow fields.

observation [9] of constant bubble diameter in the region of low site density was claimed [3, 5] to be due to the negligible mutual influence between bubbles in that region. The flow field in the influence domain in this region of nucleate boiling can be simulated by a simple familiar flow pattern.

First, the discontinuous bubble column can be replaced by a continuous vapor column. This is reasonable in view of the fact that the intermittent bubble action on the flow field in the liquid phase is decaying rapidly along the radial direction owing to the viscous damping and the inertia of the liquid flowing upwards. Recent experimental results [3] on the ebullition cycle in nucleate boiling indicate that the diameter of the area affected by the intermittent action is about twice the bubble diameter. This affected area is thus considerably smaller than the influence domain of one bubble. The rapid decaying action was also implied by the observation [7] of relatively slow movement of liquid in the boundary layer as compared to the large interfacial velocity of growing bubbles. Neglecting this intermittent character and the small diameter of the bubble as compared to that of the influence domain makes the flow field in nucleate pool boiling a simple, inverted, stagnation flow pattern as shown in Fig. 1(b).

In the second region where the site density is large, the mutual influence between bubbles becomes significant. The bubble diameter was observed [9] to be decreasing with an increase of site density. The intermittent action cannot be neglected in this region because of the small influence domain of one bubble, and the flow and heat transfer in the influence domain are thus of transient character. A description of this transient flow and heat-transfer characteristics requires firstly a commanding knowledge of the bubble dynamics. The subject of bubble dynamics has been greatly advanced in recent years, but it is still not sufficiently developed to predict the flow and heat transfer in the influence domain. In view of the fact that the flow in the wake of the bubble is an inverted stagnation flow, it is felt that the hydrodynamic model shown in Fig. 1(b) might serve as an approximation to the actual flow field in this region as well.

The proposed hydrodynamic model of inverted stagnation flow possesses the same velocity distribution, except for a change of sign in velocities, as the well-known stagnation flow against a wall [10]. Since the direction of flow will not affect the temperature distribution, the same heat-transfer result is expected as in the case of stagnation flow. While the stagnation flow is one of a few cases for which an exact solution of the Navier–Stokes equations is known, its solution indicates a boundary-layer character near the wall. The results of laminar stagnation flow and heat transfer as tabulated in Table 1 show a striking similarity to those of boundary-layer flow over a flat plate. This is, of course, no surprise for they are two special cases of the general wedge flow [10]. The present model is thus of a boundary-layer type and is similar to the one advanced by Zuber [5].

One comment on a possible model of jet type is noteworthy here. At first glance, since the rising bubble column is emerging like a jet from

Table 1. Similarity between stagnation flow and flat-plate boundary-layer flow

	Axisymmetrical stagnation flow		Flat-plate boundary-layer flow
	[10] $u_{\infty} = \alpha x$	(3)	[10] u_{∞} = constant
Laminar			
Flow	[10] $(\delta/x) = 2{\cdot}44\ Re_x^{-0{\cdot}5} = 2{\cdot}44\ (\nu/u_{\infty}x)^{0{\cdot}5}$	(4)	[10] $(\delta/c) = 5{\cdot}0\ Re_x^{-0{\cdot}5}$
Heat transfer	[12] $Nu_x = 1{\cdot}32\ Pr^{0{\cdot}33}\ Re_x^{0{\cdot}5}$	(5)	[11] $Nu_x = 0{\cdot}332\ Pr^{0{\cdot}33}\ Re_x^{0{\cdot}5}$
Turbulent			
Flow	[13] $(\delta/x) = 0{\cdot}08\ Re_x^{-0{\cdot}2}$	(6)	[11] $(\delta/x) = 0{\cdot}376\ Re_x^{-0{\cdot}2}$
Heat transfer	———		[11] $Nu_x = 0{\cdot}029\ Pr^{0{\cdot}33}\ Re_x^{0{\cdot}8}$

a round orifice, the induced flow pattern in the liquid phase might assume a form of hydrodynamic jet, which is again of a boundary-layer type [10]. This conjecture, however, is erroneous. The flow pattern in a hydrodynamic jet is developed through the mixing action, i.e. jet mixing, between the fluid of the emerging jet and the fluid surrounding it. This is obviously not the case in nucleate boiling where the emerging vapor jet does not mix with the neighboring liquid.

THERMAL BOUNDARY-LAYER THICKNESS

Ingenious measurements of thermal boundary-layer thickness in nucleate pool boiling were made by Yamagata *et al.* [7] by application of optical techniques. The temperature variation in the liquid over the heating surface resulted in a deflection of the parallel rays of light which were set to pass through the liquid. The thermal boundary-layer thickness can then be calculated from this deflection. From the measured results of thermal boundary-layer thickness and heat-transfer coefficient, two distinct kinds of behavior were observed, as shown in Fig. 2, and were attributed to the transition of flow field from laminar to turbulent. These measurements were made at very low heat fluxes and the observed transition occurred in the region of isolated bubbles, as discussed above.

Consider first the thermal boundary-layer thickness in laminar flow. The laminar flow field is expected to exist when the site density and consequently the stirring action are sufficiently small. Since analytical results for the flow and heat transfer in axisymmetrical laminar stagnation flow have been obtained, the predicted variation of heat-transfer coefficient with respect to thermal boundary-layer thickness can be made to compare with the measured results. Substitution of (3) into (4) in Table 1 gives

$$\delta = 2{\cdot}44\ (\nu/a)^{0{\cdot}5} \tag{7}$$

where a is the characteristic constant of stagnation flow. By use of the approximate relation in boundary-layer theory,

$$\delta/\delta_{th} = Pr^{0{\cdot}33}, \tag{8}$$

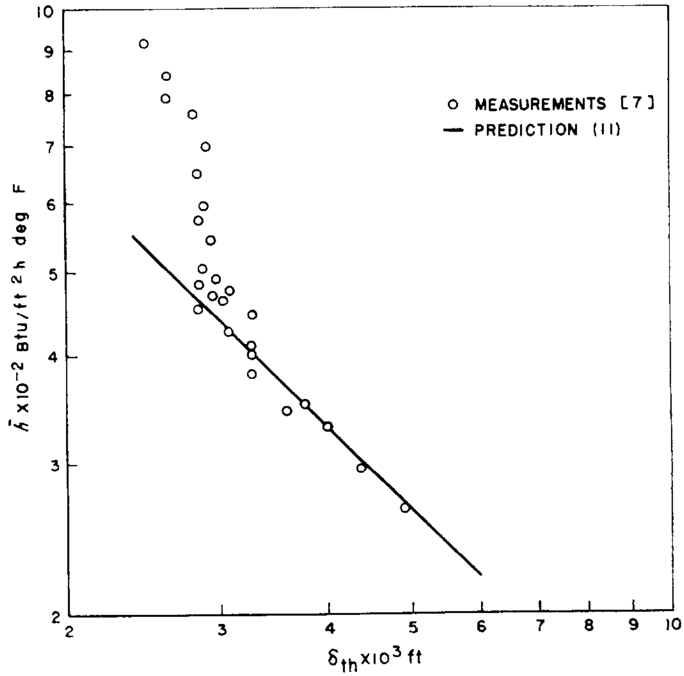

FIG. 2. Variation of boiling heat-transfer coefficient with thermal boundary-layer thickness.

the thermal boundary-layer thickness is given by

$$\delta_{th} = 2{\cdot}44\,(Pr)^{-0{\cdot}33}\,(\nu/\alpha)^{0{\cdot}5}. \tag{9}$$

The above relation indicates that δ_{th} is independent of the spatial co-ordinate so that the thermal boundary layer is uniform over the heating surface, as shown in Fig. 1(b).

Before the heat-transfer coefficient is evaluated, two points should be mentioned about the heat-transfer formula (5) in Table 1. Originally Sibulkin [12] obtained his formula as

$$\overline{Nu} = 1{\cdot}32\,Pr^{0{\cdot}4}\,Re_x^{0{\cdot}5}. \tag{10}$$

The exponent of Prandtl number in (10) was obtained by comparing numerical heat-transfer results computed at different Prandtl numbers. For most gases the Prandtl number is less than unity and the exponent 0·4 gives better agreement in the computed results. In boiling, however, the fluid is a liquid and, for liquids, exponent 0·33 is chosen, since it provides better agreement in the range of Prandtl number from 1 to 10. Secondly the heat-transfer relation in (10) is based on the average heat-transfer coefficient over an area enclosed by a circle of radius x. The change to local heat-transfer relation in (5), however, is legitimate because of the constant local heat-transfer coefficient along the surface, as implied by the existence of a uniform thermal boundary layer.

Appropriate combination of (3), (5) and (9) gives

$$\bar{h}\,\delta_{th} = h_x\,\delta_{th} = 3{\cdot}22\,k. \tag{11}$$

For water at around 220°F, $\bar{h}\,\delta_{th} = 1{\cdot}27$ Btu/h ft °F, the agreement with the experimental data [7] as indicated in Fig. 2 is surprisingly good in view of the idealization of the physical system.

When the site density increases, the stirring action created by bubbles becomes strong enough to cause a transition from laminar to turbulent flows, as indicated by a sudden change of the relationship in Fig. 2. From the analytical formula (6) in Table 1, the turbulent thermal boundary-layer thickness is found to vary with the three-fifths' power of radial co-ordinate x, and thus there no longer exists a uniform thermal boundary-layer thickness over the heating surface. Since the measurement of thermal boundary-layer thickness [7] is based on the deflection of light, the optically measured value indicates only the maximum local thickness in the turbulent region. This local value therefore cannot be meaningfully correlated with the average heat-transfer coefficient over both the laminar and turbulent regions as indicated in the left part of Fig. 2. The lack of information of transition in stagnation flow makes any analytical calculation of average heat-transfer coefficient impossible. As an approximation, the laminar heat-transfer relation will be used for the entire influence domain of one bubble.

HEAT-TRANSFER RESULTS

With the proposed mechanism of laminar stagnation flow, a correlation for the heat transfer in nucleate pool boiling can be formulated rather easily. Central to the problem, however, is the interpretation of the stagnation flow constant α in nucleate boiling. The magnitude of the constant α determines the strength of stagnation flow or in the present case the pumping action created by bubbles. As the nucleation site density increases, the area of influence domain decreases according to (2); the pumping action and with it the magnitude of constant α increases. This rough physical argument may serve to give a physical insight into the variation of constant α with respect to the site density; however, an alternative argument based on dimensional analysis yields a more complete relation.

According to the Buckingham π-theorem in dimensional analysis [14], the number of independent dimensionless groups that can be formed by combining the physical variables in a problem is equal to the total number of these physical quantities minus the number of primary dimensions involved. The physical variables in the present hydrodynamic model are α, ν and n, the primary dimensions involved are length and time, and consequently there exists only one independent dimensionless group, i.e.

$$(\alpha/n\nu) = \beta \tag{12}$$

where β is a dimensionless constant to be determined from experimental data. Since the flow field in the present model is fully described by the above three variables, the constant β is a universal constant. Substitution of (12) into (9) and comparison with Yamagata and his co-workers' data for boiling of water yield:

$$\beta = 2150. \tag{13}$$

With the constant β determined, a prediction of heat-transfer results in nucleate pool boiling is readily given by appropriate manipulation of (5), (12) and (13):

$$\bar{h} = 61{\cdot}3\, Pr^{0{\cdot}33}\, k\, n^{0{\cdot}5} \tag{14}$$

or

$$q = 61{\cdot}3\, Pr^{0{\cdot}33}\, k\, n^{0{\cdot}5}\, \Delta T. \tag{15}$$

Existing measurements of heat transfer and nucleation site density are shown in Fig. 3, where the heat-transfer coefficients for boiling of different liquids and for different surface conditions show a similar variation of site density, i.e.

$$\bar{h} \sim n^{\gamma}. \tag{16}$$

The numerical constant γ lies in the range of 0·3 and 0·5 as compared to the predicted value of 0·5. A closer comparison between the predicted result and experimental measurements is shown in Fig. 4, where boiling data of solutions are not included since some physical properties of these solutions such as Prandtl number and thermal conductivity were not known.

In the region of low site density, the heat-transfer coefficient appears to be independent of site density ($h \sim n^0$). This is attributed to the predominating influence of free convection in this region. This very reason also explains the fact that experimental results for large site density indicate a smaller exponent of site density than predicted in the heat-transfer relation. The consistent large deviation of boiling data of carbon tetrachloride and n-hexane from the prediction implies the inadequacy of the physical model in those cases. Since these scattering points still indicate the same variation of heat transfer result with site density, it is felt that the predicted functional dependence of heat-transfer result on physical properties does not represent the right dependence. Improvement could be made if the exponent of Prandtl number is

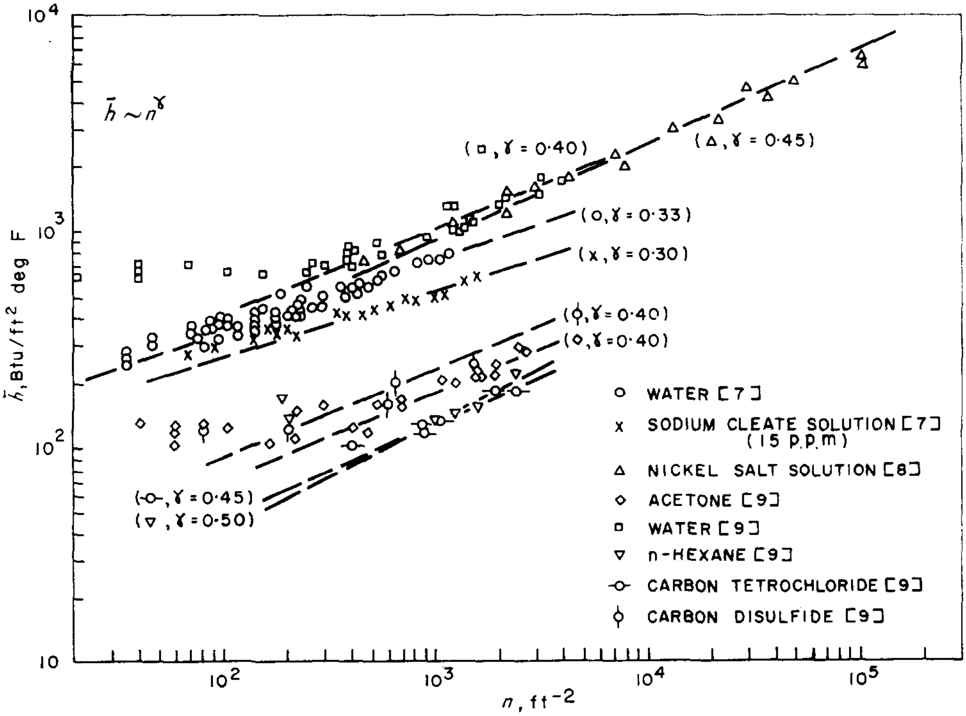

FIG. 3. Variation of boiling heat-transfer coefficient with active site density.

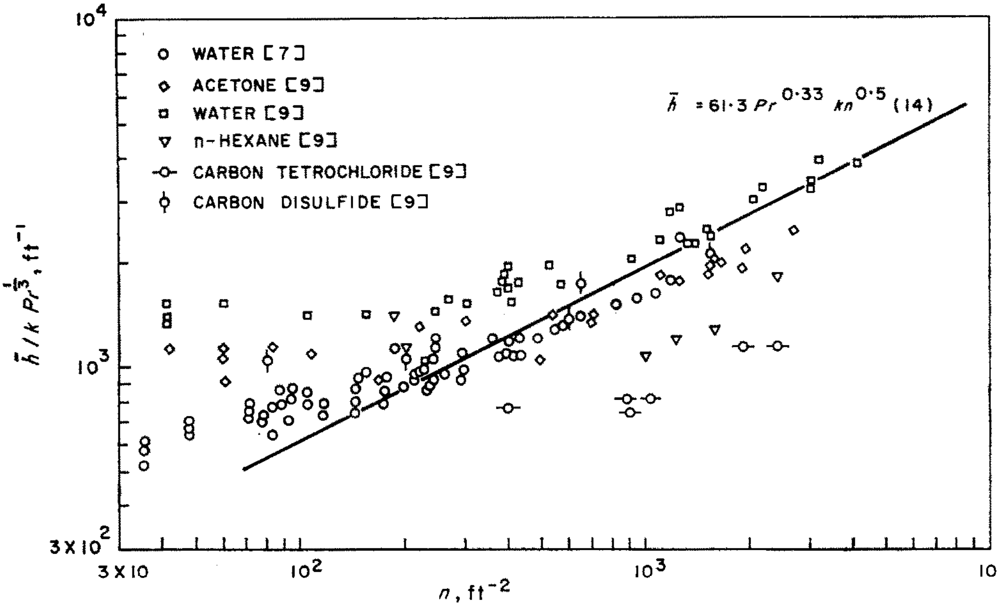

FIG. 4. Comparison between the predicted heat-transfer relation and experimental data.

reduced in (14) or (15), but it is not done here since insufficient justification can be shown for such an adjustment.

It is interesting to note that an empirical correlation of boiling data similar to (14) has been obtained by Kurihara and Meyers [8]. In their correlation, the properties of the vapor phase were included and better agreement with experimental results than the present prediction was obtained. This may be interpreted as meaning that considerable error is introduced in the present model through the neglect of vapor phase. An analytical consideration of the effect of vapor phase seems to be the next logical step to improve the present simple hydrodynamic model.

If, in the case of sub-cooled boiling, the bubbles do not collapse in the vicinity of the heating surface, a stagnation flow pattern will still exist, and the predicted relation (14) might also serve as an approximation with an appropriate change of the empirical constant, 61·3. In general, bubbles depart from the surface during the pool boiling of water at atmospheric pressure, when the bath temperature is greater than 180° or 190°F.

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Résumé—L'auteur propose un modèle hydrodynamique d'écoulement au point d'arrêt pour étudier l'ébullition nucléée saturée sur une plaque plane. A l'aide des résultats calculés pour un écoulement de révolution au point d'arrêt, il obtient une relation entre le coefficient d'échange thermique et l'épaisseur de la couche limite thermique induite par les bulles qui montent; cette relation est bien vérifiée par les résultats des mesures dans la région des faibles flux thermiques. La relation proposée s'écrit

$$q = c\,Pr^{0,33}\,k\,n^{0,5}\,\Delta T$$

où c est une constante numérique égale à 61,3 (détermination faite à partir des données sur l'ébullition de l'eau). Les données concernant l'ébullition de la plupart des liquides vérifient bien cette relation.

Zusammenfassung—Die Vorgänge an einer ebenen Oberfläche beim Blasensieden mit Verdampfung können durch das hydrodynamische Modell der Staupunktströmung veranschaulicht werden. Aus den Ergebnissen der achsensymmetrischen Staupunktströmung folgt eine Beziehung zwischen dem Wärmeübergangskoeffizienten und der Dicke der thermischen Grenzschicht, wie sie von aufsteigenden Blasen herrührt. Im Bereich kleiner Wärmestromdichten zeigt sich dabei gute Übereinstimmung mit Messergebnissen. Die Wärmestromdichte ergibt sich zu

$$q = c\,Pr^{0,33}\,k\,n^{0,5}\,\Delta T$$

mit der Konstanten $c = 61{,}3$, die aus Daten für siedendes Wasser gewonnen wurde. Die gefundenen Beziehungen stimmen mit denen der meisten anderen Flüssigkeiten überein.

Аннотация—Предлагается гидродинамическая модель заторможенного потока для насыщенного пузырькового кипения на плоской поверхности. Путём аналитического расчёта указанной задачи для осесимметричного заторможенного потока получено соотношение межлу коэффициентом теплопереноса и толщиной теплового пограничного слоя, вызванной поднятием пузырьков. Отмечается хорошее совпадение результатов измерения для режимов с малыми тепловыми потоками. Результаты вычисления теплопереноса даны в виде соотношения

$$q = c\,Pr^{0,33}\,k\,n^{0,5}\,\Delta T$$

где c—постоянная, равная 61,3, определена на основе опытных результатов по кипению воды. Найдено, что данное соотношение согласуется с данными по кипению большинства жидкостей.